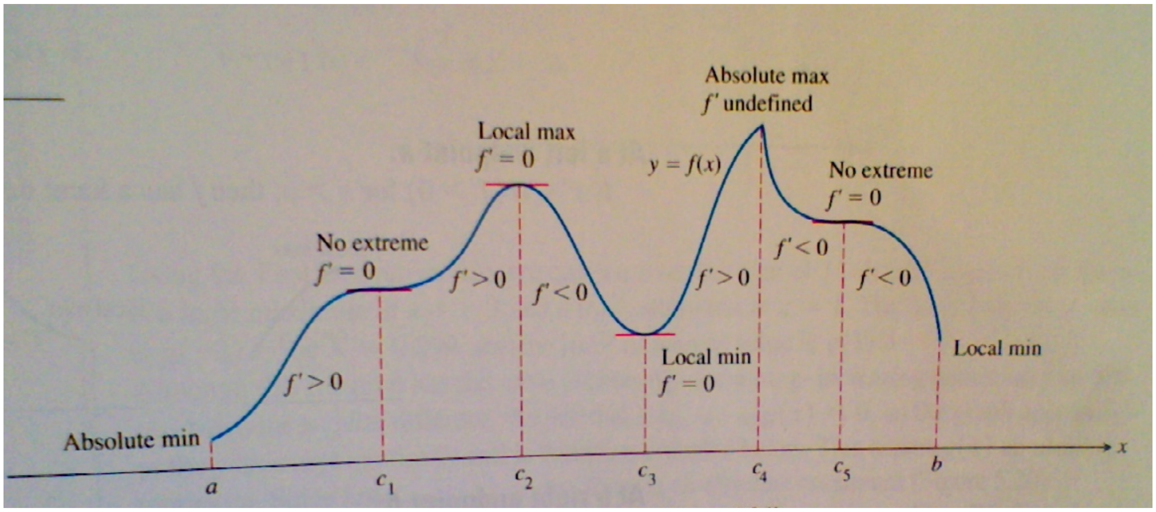
Connecting f' and f'' with the graph of f

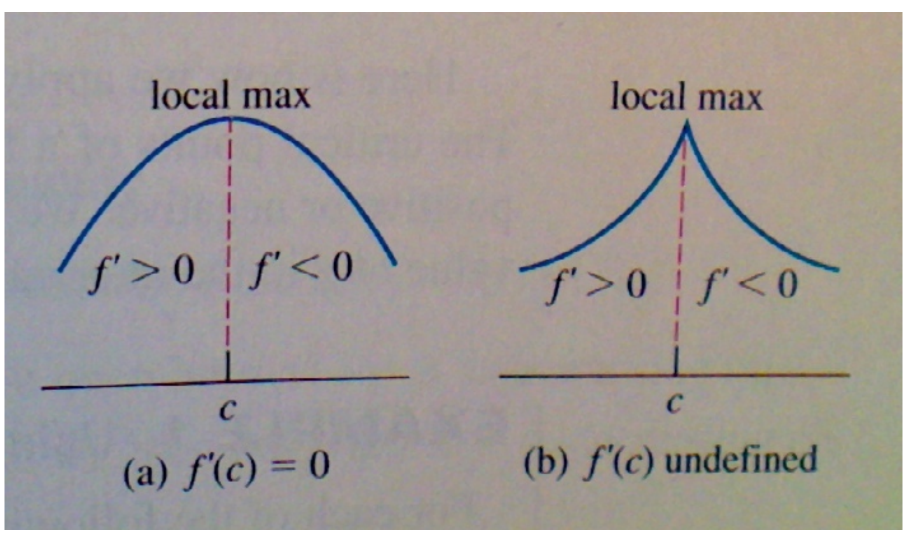


Theorem-First Derivative Test for Local Extrema

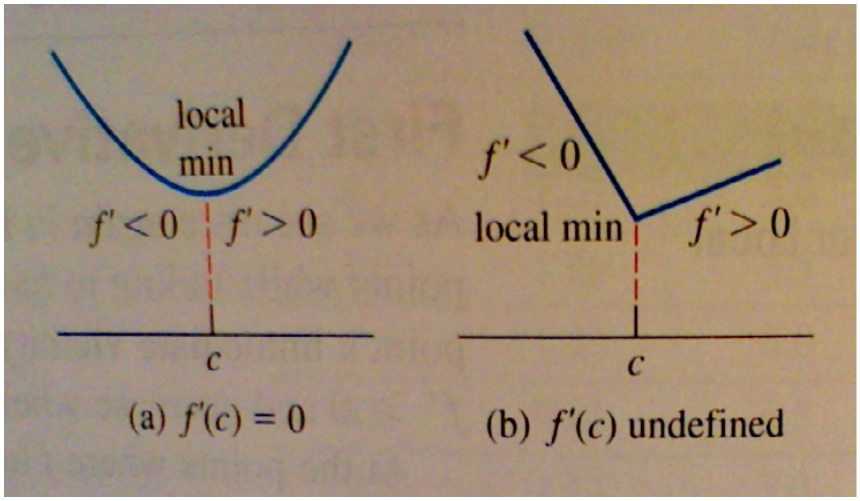
-The following test applies to a continuous function f.

At a critical point c:

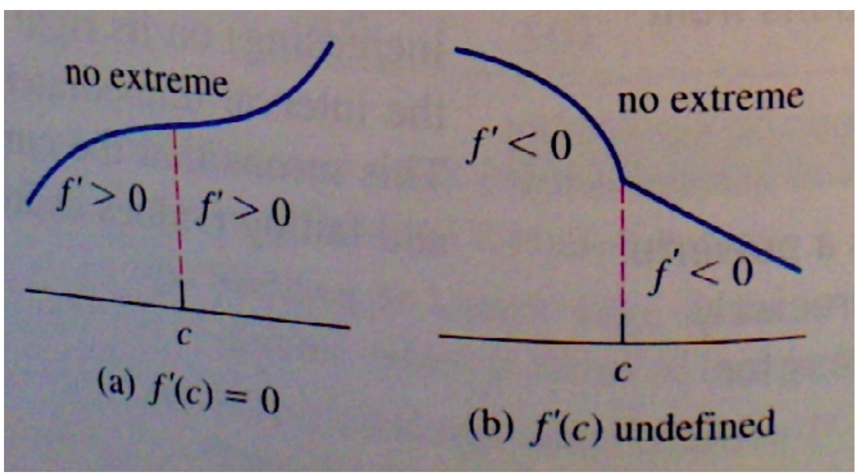
1) If f' changes sign from positive to negative at c, $\left(f' > 0 \text{ for } x < c \text{ and } f' < 0 \text{ for } x > c\right)$, then f has a **local max** value at c.



2) If f′ changes sign from negative to positive at c, $\left(f' < 0 \text{ for } x < c \text{ and } f' > 0 \text{ for } x > c\right)$, then f has a **local min** value at c.



3) If f′ does not change sings at c, (f′ has the same sign on both sides of c), then f has no extreme values at c.



Example

Find the critical points of $f\left(x\right) = x^3 - 12x - 5$. Find the functions local max and min values. Identify the intervals where it is increasing and decreasing.

-Since f is continuous and differentiable the critical points only occur at the zeros of f′.

$f'(x) = 3x^2 - 12 = 0$

$x^2 = 4$

$x = -2, 2$

-The zeros of f' can be used to partition the interval.

Intervals	$-\infty < x < 2$	$-2 < x < 2$	$2 < x < \infty$
Sign of f'	+	-	+
Behavior of f	Increasing	Decreasing	Increasing

-We can see from the table there is a local max at
$x = -2$ and min at $x = 2$.

-The local max value is $f(-2) = 11$ and the local min value is $f(2) = -21$.

-There are no absolute extremea.

-The function increases on $(-\infty, -2]$ and $[2, \infty)$ and decreases on the interval $[-2, 2]$.

Example

$f(x) = (x^2 - 3)e^x$

$f'(x) = (x^2 - 3) \bullet \frac{d}{dx} e^x + \frac{d}{dx}(x^2 - 3)e^x$

$= (x^2 - 3)e^x + (2x)e^x$

$= (x^2 + 2x - 3)e^x$

-Since e^x is never 0, $f'(x)$ is 0 iff

$$x^2 + 2x - 3 = 0$$

$$(x+3)(x-1) = 0$$

$$x = -3, 1$$

Intervals	$x < -3$	$-3 < x < 1$	$1 < x$
Sign of f'	+	-	+
Behavior of f	Increasing	Decreasing	Increasing

Local max at $x = -3$

Local min at $x = 1$

Increases on $(-\infty, -3]$ and $[1, \infty)$ and decreases on $[-3, 1]$